A State-of-the-Art Survey of Due Date Assignment and Scheduling Research: SLK, TWK and Other Due Date Assignment Models

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N° 3537
Octobre 1998

THÈME 4

Rapport de recherche

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ABSTRACT

This paper is a review of the results on the due date assignment and scheduling problems in the static deterministic case. In the problems under consideration, the objective is to find optimal values of the due dates and the related optimal schedule so that to minimize a given criterion based on the due dates and the completion times of jobs. The problems with due date determination have received considerable attention in the last ten years due to the introduction of new methods of inventory management such as Just-In-Time systems. According to the Just-In-Time concept jobs are to be completed neither too early nor too late which leads to the problems with non-regular measure of performance that includes earliness and tardiness costs. The due date assignment models where due dates depend on the jobs' processing times or on the positions of the jobs in the schedule are considered. The results on algorithms and complexity of the due date assignment and scheduling problems are summarized.

KEYWORDS

Due date assignment, Scheduling, State-of-the-art, Manufacturing, Complexity.

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Etat de l'art sur les recherches dans le domaine de l'affectation des délais et de l'ordonnancement: SLK, TWK et différents autres modèles

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RESUME

Ce papier a pour objectif de faire l'état de l'art dans le domaine de l'affectation et de l'ordonnancement par rapport aux délais dans le cas statique et déterministe. Pour les problèmes que nous examinons, l'objectif est de trouver les délais optimaux et les ordonnancements correspondants qui minimisent des critères fonctions des délais et des instants de fin de fabrication. Les problèmes dont l'objectif est de déterminer les délais optimaux ont reçu une attention particulière au cours des dernières années du fait de l'apparition de nouveaux concepts de gestion comme, par exemple, le concept de " Juste-A-Temps". Ce concept exige de terminer les fabrications ni trop tôt, ni trop tard, ce qui conduit à des problèmes pour lesquels les mesures de performance sont non-régulières et font intervenir les retards et les avances de fin de fabrication. Les modèles dans lesquels les délais dépendent des temps de fabrication ou de la position des tâches dans l'ordonnancement sont examinés. Les résultats sur la nature des algorithmes et leur complexité sont résumés dans des tableaux récapitulatifs.

MOTS CLEFS

Affectation des délais, Ordonnancement, Etat de l'art, Fabrication, Complexité.

A STATE-OF-THE-ART SURVEY OF DUE DATE ASSIGNMENT AND SCHEDULING RESEARCH: SLK, TWK AND OTHER DUE DATE ASSIGNMENT MODELS

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The Just-In-Time (JIT) concept in industry according to which jobs are to be completed neither too early nor too late, gave rise to scheduling research with non-regular measure of performance (for the review, see Baker and Scudder, 1990). The objective of minimizing earliness and tardiness costs is an example of such non-regular performance measure as opposed to a regular measure which is a non-decreasing function of the jobs' completion times. The increasing interest in search of optimal decisions in JIT systems stimulated the development of due date determination methods for the problems with non-regular measure of performance. As a result of growing attention to the due date determination problems is a wide literature dealing with the due date assignment and scheduling problems. We aim at providing a review of the results which have been published in this area after the survey of Cheng and Gupta (1989) who illustrate the results obtained up to the end of 1980's.

In this paper, which may be regarded as the second part of Gordon, Proth and Chu (1998), where the common (CON) due date determination model was observed, we consider the SLK, TWK, NOP and PPW due date determination methods. The definitions of the SLK, TWK and other due date assignment methods are given in Gordon, Proth and Chu (1998) and will be repeated in the corresponding sections of the paper. Besides, we consider the positional due dates, also known in the literature as the generalized due dates, where the assignment of a given set of due dates to the jobs occurs. As in our previous paper, we focus on the static and deterministic scheduling problems. We consider, mainly, the single machine model, since there are quite few results for multiple machines.

The following notations are used in the rest of the paper.

If d_j, C_j, E_j and T_j denote respectively the due date, the completion time, and the earliness and tardiness of job j, then

$$E_j = max\{0, d_j - C_j\} \text{ and } T_j = max\{0, C_j - d_j\}. \qquad (1)$$

Lateness L_j of job j is defined as $L_j = C_j - d_j$. Hence, $E_j = max\{0, -L_j\}$ and $T_j = max\{0, L_j\}$.

The paper is organized as follows. In Section 1, we consider the SLK due date assignment model. In Sections 2 and 3, respectively, TWK/NOP and PPW due date assignment are considered. Section 4 is devoted to the positional due dates.

1. SLK DUE DATES

Let $N = \{1, 2, ..., n\}$ be a set of jobs to be processed on a single machine with given release dates r_j and processing times p_j, $j=1,...,n$. As usually, it is assumed that machine can handle at most one job at a time. For a given schedule S, let C_j be a completion time of job j. Let E_j and T_j be the earliness and the tardiness of job j defined by (1), and let L_j be the lateness of this job.

The SLK due date assignment method determines due dates based on the common *slack* q which is added to the sum of the release dates and processing times of individual jobs: $d_j = r_j + p_j + q$, $j=1,...,n$. When all jobs are available simultaneously at time zero, $d_j = p_j + q$. This kind of due date model reflects the situation when the product facility is itself responsible for assigning due dates, and the decision maker assigns due dates by estimating job flow times.

1.1. Assume that set N is partially ordered. A partial order $\rightarrow$ on N imposes constraints on the order in which the jobs from N can be processed: $i \rightarrow j$, $i, j \in N$, implies that j cannot start before i is finished. Due date d_j is assigned as $d_j = r_j + p_j + q$, where q is a slack allowance common among the jobs.

Assuming that preemption is allowed (the processing of a job may be interrupted and resumed later), let us consider the problem of finding an optimal schedule S and an optimal slack allowance q that jointly minimize a penalty function given by

$$f(q,S) = \gamma q + max\{T_j \mid j \in N\},$$

where $\gamma \geq 0$ is the cost per time unit of slack.

To obtain a schedule that minimizes $f(q,S)$ for any given q, we may use an algorithm proposed by Gordon and Tanaev (1983) and Baker *et al.* (1983) for the single machine preemptive scheduling to minimize maximum cost subject to given precedence constraints and release dates (that is, an algorithm for the problem $1|pmtn, prec, r_j|f_{max}$ according to the notations of Lawler *et al.*, 1993). Let us denote this algorithm by Algorithm GT-BLLR.

Gordon (1993) showed that the schedule obtained by applying Algorithm GT-BLLR is optimal independently of the value of q, and that the optimal slack $q*$ is equal to zero, if $\gamma > 1$, to $C_l - r_l - p_l$ if $0 < \gamma < 1$, or may be any value in the intervals $[C_l - r_l - p_l, \infty)$ if $\gamma = 0$ and $[0, C_l - r_l - p_l]$ if $\gamma = 1$. Here, l is the job with maximal tardiness in the optimal schedule. The complexity of an algorithm which leads to an optimal schedule is $O(n^2)$ in the general case of precedence constraints and $O(n \log n)$ in the case when the graph representing the precedence constraints is a tree.

The special case of the above problem when all release dates are equal to zero and jobs are independent from each other was considered by Cheng (1989) and Alidaee (1991). In this case, the optimal schedule is non-preemptive and is defined by the SPT (shortest processing time) order. The optimal value of q is obtained as in the general case of the problem (it is sufficient to substitute $C_{[n-1]}$ for $C_l - r_l - p_l$ in the above formulas).

1.2. Later on, throughout this section, we assume that all jobs in N become available for processing simultaneously (hence, SLK due dates are assigned as $d_j = p_j + q$), and that job preemption is not permitted. In this case, the schedule is defined by the sequence σ of jobs. Let us consider the total weighted earliness and tardiness (TWET) problem with symmetric weights (see **1.5** in Gordon, Proth and Chu, 1998), that is, the problem which consists in minimizing the objective function

$$f(q,\sigma) = \sum_{j=1}^{n} \alpha_j (E_j + T_j) = \sum \alpha_j |C_j - d_j|,$$

where $\alpha_j > 0$ is a weight of job j.

Gupta, Bector and Gupta (1990) and Karacapilidis and Pappis (1995) noted that CON and SLK due date assignment methods for this problem have similar properties followed from their resembling linear programming (LP) formulation. The LP formulation for the CON method is (here, we use the notation $[j]$ for the jth job in the sequence σ, and d is a common due date):

$$\text{minimize } \sum_{j=1}^{n} \alpha_{[j]} \left| C_{[j]} - d \right| = \sum_{j=1}^{n} \alpha_{[j]} (E_{[j]} + T_{[j]})$$

$$\text{subject to } d + T_{[j]} - E_{[j]} = C_{[j]},$$

$$d, E_{[j]}, T_{[j]} \geq 0, j=1,\dots,n,$$

$$\text{where } E_{[j]} = d - C_{[j]},\ T_{[j]} = C_{[j]} - d, \text{ and } E_{[j]} T_{[j]} = 0.$$

Define the waiting time for job $[j]$ in σ by $W_{[j]} = C_{[j]} - p_{[j]} = C_{[j-1]}$, for $j=2,3,\dots,n$, and $W_{[1]} = 0$. Then, the LP formulation for the SLK method is:

$$\text{minimize } \sum_{j=1}^{n} \alpha_{[j]} \left| W_{[j]} - q \right| = \sum_{j=1}^{n} \alpha_{[j]} (E_{[j]} + T_{[j]})$$

$$\text{subject to } q + T_{[j]} - E_{[j]} = W_{[j]},$$

$$q, E_{[j]}, T_{[j]} \geq 0, j=1,\dots,n,$$

$$\text{where } E_{[j]} = q - W_{[j]},\ T_{[j]} = W_{[j]} - q, \text{ and } E_{[j]} T_{[j]} = 0.$$

The structural similarity between both LP formulations suggests that the solution approaches can be also similar. Really, Gupta, Bector and Gupta (1990) showed that the symmetric-weighted TWET problem with the SLK due date assignment has a property similar to a property of the problem with CON due dates (see Property 5 of Section **1.5** in Gordon, Proth and Chu, 1998). They proposed a polynomial-time algorithm in the case of $\alpha_j = 1$, $j=1,\dots,n$, similar to the algorithm known for the CON due dates.

The above mentioned property may be formulated as follows.

Property 1. There exists an optimal schedule which is:

a) V-shaped in the sense that the jobs in set E (non-tardy jobs) are sequenced in the non-increasing order of the ratio p_j/α_j and followed by the jobs of set T (tardy jobs) in the non-decreasing order of the ratio p_j/α_j,

b) the optimal slack q^* coincides with the waiting time of the last non-tardy job in an optimal sequence, and

c) $\sum_{j \in T} \alpha_j \leq \sum_{j \in E} \alpha_j$.

For the same problem with $\alpha_j = 1$, Karacapilidis and Pappis (1993, 1995) proposed an algorithm to find an optimal slack and the set of all optimal sequences (each optimal sequence is found in $O(n \log n)$ time). They showed that alternative optima derived from the application of the SLK method can be easily found if the alternative optima derived from the application of the CON method are known.

Oguz and Dincer (1994) proved NP-hardness of the restricted variant of the problem with $\alpha_j = 1$, i.e., when $q < \delta$, where $\delta = p_1 + p_3 + ... + p_{n-1}$ if n is even, and $\delta = p_2 + p_4 + ... + p_{n-1}$ if n is odd, assuming that the processing times are indexed in non-decreasing order. The result follows from Hall, Kubiak and Sethi (1991).

Note that in general case, the TWET problem with different due dates is NP-hard, even if $\alpha_j = 1,\ j = 1,...,n$ (Garey, Tarjan and Wilfong, 1988). In case of SLK due dates and different α_j, Gupta, Bector and Gupta (1990) proposed a heuristic procedure.

Pappis and Adamopoulos (1993) considered the problem with the objective function $\sum_{j=1}^{n}(\alpha_j E_j + \beta_j T_j)$ in case of $\alpha_j = \lambda p_j^a$, $\beta_j = \lambda p_j^b$, where $\lambda > 0$, and parameters a and b are non-negative integers. They proposed similar algorithms for the CON and SLK models. For the same problem with SLK due date assignment, Adamopoulos and Pappis (1996a) proposed branch-and-bound algorithms for solving the problem in the following cases: 1) $a = b > 1$, 2) $a = 1$, $b > 1$ or $a > 1$, $b = 1$, 3) $a > 1$, $\beta_j = 1$ or $b > 1$, $\alpha_j = 1$, and 4) $a = 1$, $\beta_j = 1$ or $b = 1$, $\alpha_j = 1$, for all j. In the second and the fourth cases, where the optimal solution is not unique, the set of all the alternative optimal sequences is derived.

Kahlbacher (1993) proved that the following problems for CON and SLK models are equivalently solvable under special conditions on the functions g and h and on the due dates: a) the problem to minimize the objective function $\sum_{j=1}^{n} g(C_j - d_j)$ for the CON due date assignment model with due dates $d_j = d$ for all jobs j, b) the problem to minimize the objective function $\sum_{j=1}^{n} h(C_j - d_j)$ for the SLK due date assignment model with due dates $d_j = p_j + q$ for all jobs j. The conditions to meet are the following: $q = \sum_{j=1}^{n} p_j - d$; $h(x) = g(-x)$, and $g(x)$ is unimodal, real valued function with the following properties: 1) $g(0) = 0$, 2) $g(x_1) \geq g(x_2)$ for all $x_1 \leq x_2 \leq 0$, 3) $g(x_1) \leq g(x_2)$ for all $0 \leq x_1 \leq x_2$. The objective functions for the MAD, WSAD and MSD problems satisfy these conditions. Here, the MAD problem is the problem to minimize the mean absolute deviation of completion times with respect to the due dates $\sum_{j=1}^{n}|C_j - d_j| = \sum_{j=1}^{n}(E_j + T_j)$. The WSAD problem is the problem to minimize the weighted sum of the absolute deviation $\alpha \sum_{j \in E}|C_j - d_j| + \beta \sum_{j \in T}|C_j - d_j| = \sum_{j=1}^{n}(\alpha E_j + \beta T_j)$, where E and T are defined as in Property 1, and the MSD problem is the problem to minimize the mean squared deviation $\sum_{j=1}^{n}(C_j - d_j)^2 = \sum_{j=1}^{n}(E_j^2 + T_j^2)$. In particular, the NP-hardness of the MSD problem with SLK due dates follows from these results and Kubiak (1993), where the NP-hardness was proved for the CON due date model.

1.3. Let us consider the problem of finding a slack q and a sequence σ which minimize the objective function

$$f(q,\sigma) = \sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]} + \gamma q).$$

Using the similarity between the CON and SLK models and the results of Panwalkar, Smith and Seidmann (1982) for the CON model (see **1.2** in Gordon, Proth and Chu, 1998), Adamopoulos and Pappis (1996b) proposed a polynomial algorithm for the SLK model. They showed that, in any sequence, there is a job occupying position K such that any job after this position is tardy and any job before it is early. The position K is given by the smallest integer greater than or equal to $n(\beta - \gamma)/(\alpha + \beta)$. The optimal sequence is given by determining the positional coefficients and allocating the jobs in such positions that the pairwise product of positional coefficients and the jobs' processing times is minimized. This result is similar to the one obtained by Panwalkar, Smith and Seidmann (1982). The optimal q is equal to the waiting time of the Kth job in the optimal sequence: $q^* = W_{[K]} = C_{[K]} - p_{[K]}$.

1.4. Consider the problem of minimizing the penalty of earliness under the constraint of no tardy jobs.

Let the set $N = \{1,2,...,n\}$ of jobs be partially ordered (an order is given by the graph of precedence constraints). A schedule is feasible if no job is tardy and the sequence of jobs meets the precedence constraints. For a given schedule, job j is said to be *on time*, if $C_j = d_j$, and early, if $C_j \leq d_j$.

Gordon and Strusevich (1998) considered the problem of finding a value of slack q and a feasible schedule S to minimize the objective function $\varphi(F,q)$, where φ is an arbitrary non-decreasing function in both arguments and $F = F(E_1, E_2, ..., E_n)$ is a non-decreasing function with respect to each of its arguments. The function F represents penalties for the early completion of the jobs and can be viewed as the holding cost function. The function φ is considered to be non-decreasing in q so as to model the situation when due dates are to be small enough to satisfy the potential customer.

The following properties characterize feasible schedules (Gordon and Strusevich, 1998).

<u>Property 2</u>. A feasible schedule exists if and only if there exists a job m with no successors (with respect to the given order) such that

$$p(N) - p_m \leq q, \tag{2}$$

where $p(N) = \sum_{j=1}^{n} p_j$.

<u>Property 3</u>. There exists a feasible schedule such that

a) a job m that has no successors and satisfies (2) is scheduled last and is on time,

b) the other jobs are scheduled in the time interval $[d_m - p(N),\ q]$, follow a feasible sequence (with respect to the given order) and are completed strictly before their due dates.

Similar property was proved by Qi and Tu (1998) in the case of independent jobs.

Gordon and Strusevich (1998) proposed an algorithm which provides a general scheme for solving the problem. The complexity depends on the kind of partial order and on the specific form of function F.

When $F(E_1, E_2, \dots, E_n) = \sum_{j=1}^{n} E_j$, the problem is shown to be NP-hard in the strong sense for arbitrary precedence constraints. On the other hand, for the series-parallel precedence constraints and F being total weighted earliness (i.e., $F(E_1, E_2, \dots, E_n) = \sum_{j=1}^{n} \alpha_j E_j$), the problem is solvable in $O(n^2 \log n)$ (Gordon and Strusevich, 1998). Remind that a graph is called series-parallel if it can be obtained from single-vertex graphs by subsequent application of the operations of series and/or parallel compositions (see Tanaev, Gordon and Shafransky, 1994, for further details on series-parallel graphs). The crucial observation for the series-parallel precedence constraints is that any set of jobs which have one common successor should have the same set of successors. In other words, the precedence graph does not contain a Z-shaped subgraph, where Z is a four-vertex graph whose vertices 1 and 2 correspond to the upper part of the letter Z, and vertices 3 and 4 correspond to the lower part of Z:

$$\begin{tikzcd} 1 \arrow[r] & 2 \\ 3 \arrow[r] \arrow[ur] & 4 \end{tikzcd}$$

and vertices 1 and 3 have common successor 2, but vertex 4 is the successor only for vertex 3. Recognition whether the n-vertex graph is series-parallel can be done in $O(n^2)$ time (Valdes, Tarjan and Lawler, 1982).

For the independent jobs and $F(E_1, E_2,..., E_n) = \sum_{j=1}^{n} \alpha_j E_j$, Gordon and Strusevich (1998) proposed an $O(n \log n)$ algorithm, thus improving the result of Qi and Tu (1998) who proposed an $O(n^2)$ algorithm.

For F being total weighted exponential earliness penalty (i.e., $F(E_1, E_2,..., E_n) = \sum_{j=1}^{n} \alpha_j exp(\gamma E_j)$, where $\gamma \neq 0$), similar results were obtained: the problem is solvable in $O(n^2 \log n)$ time for the series-parallel precedence constraints and in $O(n \log n)$ time for the independent jobs (Gordon and Strusevich, 1998).

In the case of independent jobs, Qi and Tu (1998) proposed an $O(n \log n)$ algorithm for minimizing the objective function $\sum_{j=1}^{n} g(E_j)$, where g is a non-decreasing function.

The results are summarized in Table 1. We adopt the standard three-field notation $a|b|c$ used for scheduling problems (Lawler *et al.*, 1993), where a describes machine environment and specifies the number of machines, b describes schedule and job characteristics, and c describes the optimality criterion. We extend this notation in the following way. The notation $d_j = r_j + p_j + q$ or $d_j = p_j + q$ denotes the problem with the SLK due date assignment (d_j^{res} denotes the restricted variant of the problem). The notation $C_j \leq d_j$ denotes the problem with the condition of no tardy jobs, the notation *SP* denotes the problem with series-parallel precedence graph. As usual, the notations *pmtn*, *prec*, *tree* denote, respectively, the problem with preemption allowed, the problem with precedence constraints given by an arbitrary acyclic graph and by a tree-like graph. In the column **Complexity**, notation P denotes polynomially solvable problem. In the column **Algorithm**, the notation Heurist. denotes heuristic procedure. We do not include in the table the results related to the special cases of the problems. Taking into account the similarity between the SLK and CON due date assignment models, the results on the complexity and algorithms for the SLK model may be supplemented by some additional information from the table for the CON due date assignment and scheduling problems (see Gordon, Proth and Chu, 1998).

Table 1. SLK due date assignment and scheduling

Problem	Complexity	Algorithm
$1\|pmtn, prec, r_j, d_j = r_j + p_j + q\|\gamma q + max\, T_j$	P	$O(n^2)$ Gordon (1993)
$1\|pmtn, tree, r_j, d_j = r_j + p_j + q\|\gamma q + max\, T_j$	P	$O(n \log n)$ Gordon (1993)
$1\|d_j = p_j + q\|\gamma q + max\, T_j$	P	$O(n \log n)$ Cheng (1989), Alidaee (1991)
$1\|d_j = p_j + q\|\sum(E_j + T_j)$	P	$O(n \log n)$ Gupta *et al.* (1990), Karacapilidis&Pappis (1993, 1995)
$1\|d_j^{res} = p_j + q\|\sum(E_j + T_j)$	NP-hard Oguz&Dincer (1994)	
$1\|d_j = p_j + q\|\sum(\alpha E_j + \beta T_j + \gamma d)$	P	$O(n \log n)$ Adamopoulos& Pappis (1996)b
$1\|d_j = p_j + q\|\sum \alpha_j(E_j + T_j)$	NP-hard	Heurist. Gupta *et al.* (1990)
$1\|d_j = p_j + q\|\sum(E_j^2 + T_j^2)$	NP-hard (from Kubiak, 1993 and Kahlbacher,1993)	
$1\|d_j = p_j + q, C_j \le d_j\|\sum g(E_j)$	P	$O(n \log n)$ Qi&Tu (1998)
$1\|d_j = p_j + q, C_j \le d_j\|\sum \alpha_j E_j$	P	$O(n \log n)$ Gordon&Strusevich (1998)
$1\|d_j = p_j + q, C_j \le d_j\|\sum \alpha_j\, exp(\gamma E_j)$	P	$O(n \log n)$ Gordon&Strusevich (1998)
$1\|SP, d_j = p_j + q, C_j \le d_j\|\sum \alpha_j E_j$	P	$O(n^2 \log n)$ Gordon&Strusevich (1998)
$1\|SP, d_j = p_j + q, C_j \le d_j\|\sum \alpha_j\, exp(\gamma E_j)$	P	$O(n^2 \log n)$ Gordon&Strusevich (1998)
$1\|prec, d_j = p_j + q, C_j \le d_j\|\sum E_j$	strongly NP-hard Gordon& Strusevich (1998)	

2. TWK AND NOP DUE DATES

In the TWK due date assignment method, where TWK stands for *total work content,* the due dates are equal to a multiple of the job processing times. If we assume that all jobs are available simultaneously, the due dates are set as $d_j = kp_j$, where the constant $k > 0$ represents the common multiple which is to be determined to define the due date for each job. When the release dates are given, $d_j = r_j + kp_j$. In some papers, the TWK method includes also a non-negative processing time exponent *m*, and the due dates are set as $d_j = kp_j^m$, $j=1,...,n$, (the so called TWK-power due dates). When release dates r_j are given for each job *j*, the TWK-power due dates are set as $d_j = r_j + kp_j^m$ (but sometimes as $d_j = kp_j^m$).

In the NOP due date assignment method, the due dates are determined on the basis of the number of operations to be performed on the job: $d_j = kM_j$, where *k* is the constant to be determined and M_j is the number of operations required to complete job *j*.

Let a set of independent jobs $N = \{1, 2, ..., n\}$ be processed on a single machine with given processing times p_j, $j=1,...,n$. All jobs are supposed to be available at time zero unless otherwise stipulated. For a given schedule, let C_j, T_j and L_j be, respectively, the completion time, the tardiness and the lateness of job *j*.

2.1. For the TWK due dates defined as $d_j = kp_j$, $j=1,...,n$, Cheng (1984) (see, also, the remark of Mutlu, 1993) considered the problem of finding a value of *k* and a sequence of jobs which minimize the total squared lateness $\sum_{j=1}^{n} L_j^2$. It was shown that the optimal sequence is the SPT sequence (where jobs are placed in non-decreasing order of their processing times), and the optimal common multiple is given as

$$k^* = (\sum_{j=1}^{n} p_{[j]} \sum_{i=1}^{j} p_{[i]}) / \sum_{j=1}^{n} p_{[j]}^2,$$

where $[j]$ is the *j*th job in the SPT sequence.

For the TWK due dates defined by $d_j = kp_j^m$, $j=1,...,n$, where $m \geq 1$ is given, Cheng (1987a) and Cheng and Li (1989) considered the problem of finding a value of *k* and a sequence of jobs which minimize the sum of mean flow time and the total squared lateness

$$f(k, \sigma) = \sum_{j=1}^{n} C_j / n + \sum_{j=1}^{n} L_j^2.$$

The optimal solution is obtained by applying the SPT rule and using any non-linear search method for finding an optimal multiple k^*.

Gupta, Bector and Gupta (1990) considered the symmetric-weighted TWET problem (see **1.2**) with the TWK due dates $d_j = kp_j, j=1,...,n$. They did not propose any approaches to the solution of the problem but proved the following property which may be useful for finding a solution.

Property 4. For any specified sequence σ, there exists an optimal value k *, which coincides with the ratio of completion time and processing time of exactly one of the jobs in σ.

2.2. For the NOP due dates $d_j = kM_j$, Alidaee (1992) considered the problem of finding an optimal multiple k * and an optimal sequence σ * which minimize the objective function

$$f(k,\sigma) = \gamma k + max_{j \in N} T_j,$$

where $\gamma \geq 0$.

The following property gives an optimal ordering for any fixed value of k (Alidaee, 1992).

Property 5. Given a value of k, the earliest due date (EDD) order minimizes the objective function $f(k,\sigma)$.

Hence, the problem is reduced to the problem of finding an optimal value k * which minimizes the following objective function:

$$f(k, EDD) = \gamma k + max_{1 \leq j \leq n} \{max\{0, C_j - kM_j\}\}.$$

Alidaee (1992) proposed an $O(n)$ algorithm to solve this problem. Due to similarity of the NOP due dates $d_j = kM_j$ and the TWK due dates $d_j = kp_j$, these results are also applicable to the TWK due dates. For the problem with the TWK due dates, Cheng (1991) proposed an approach to minimize the more general objective function $\Phi(k) + \alpha\, max_{j \in N} T_j$, where $\alpha > 0$, and $\Phi(k)$ is a non-decreasing convex function.

Cheng and Gordon (1994) considered TWK-power due dates $d_j = kp_j^m$ for the problem with given release dates, a partially ordered set N of jobs and with the assumption that preemption is allowed. The problem is to find an optimal schedule S * and optimal values k * and m * that jointly minimize the objective function

$$f(k,m,S) = \Phi(k,m) + \alpha\, max_{j \in N} T_j,$$

where $\alpha > 0$, and $\Phi(k,m) \geq 0$ is a non-decreasing convex function of k and m with $\Phi(0,0) = 0$. The function Φ represents the cost of assigning the due dates. This cost increases with the due dates since customers, in general, prefer quick delivery and are reluctant to wait for a long period.

The following property allows to find an optimal schedule in $O(n^2)$ time (Cheng and Gordon, 1994).

Property 6. The schedule obtained by applying Algorithm GT-BLLR (see **1.1**) is optimal independently of the values of k and m.

Hence, the completion times for the optimal schedule $S*$ do not depend on the values of k and m but only on r_j, p_j and the jobs' precedence constraints. However, the job which will have the maximal tardiness in $S*$ depends on the values of k and m.

2.3. For the TWK-power due dates defined by $d_j = kp_j^m$, $j=1,...,n$, where $m \geq 1$ is a given exponent and k is the due date multiplier which is a decision variable, Cheng (1987b) considered the single machine due date determination problem for a given schedule with the objective function

$$f(k) = \sum_{j=1}^{n} (\gamma k + |C_j - d_j|) ,$$

where γ is the cost per unit value of k. The problem can be formulated as

$$\min\{f(k) | k \geq 0\},$$

where k is the only decision variable involved.

Cheng (1987b) reformulated this problem in terms of linear programming (LP) problem and proposed an $O(n^2)$ algorithm, the validity of which was proved on basis of the dual of the LP problem. Van de Velde (1990) gave a simpler and faster algorithm based upon strictly primal arguments, requiring $O(n \log n)$ time.

3. PPW DUE DATES

The PPW due date assignment model, where PPW stands for *processing-plus-wait*, combines the CON, SLK and TWK due dates in one model, in which the due dates are linear function of the job processing times.

In the PPW model, $d_j = kp_j + q$ for simultaneously available jobs, where $k \geq 0$ is a common multiplier, and q is a slack allowance which may be negative. When release dates are given for each job j, the due dates are set as $d_j = r_j + kp_j + q$, or sometimes still as $d_j = kp_j + q$. The optimal due date assignment consists in finding the optimal values of k and q.

Let, as before, N be a set of n jobs with processing times p_j, $j=1,...,n$. For a given schedule, let C_j, E_j, T_j and L_j be, respectively, the completion time, the earliness, the tardiness and the lateness of job j.

3.1. Assume that the simultaneously available jobs (i. e., jobs having the same release date) are to be processed on a single machine starting at time zero without preemption. In this case, the schedule is defined by a sequence σ of jobs. The PPW due dates are set as $d_j = kp_j + q$, $j=1,...,n$.

Kahlbacher and Cheng (1995) considered the problem of finding the optimal values of parameters k and q and an optimal sequence to minimize the mean squared deviation (MSD) of completion times with respect to the due dates (or, which is the same, the mean squared lateness):

$$f(k,q,\sigma) = (1/n)\sum_{j=1}^{n}(C_j - d_j)^2 = (1/n)\sum_{j=1}^{n}(C_j - kp_j - q)^2.$$

For a fixed job sequence, Kahlbacher and Cheng (1995) found the optimal parameters k^* and q^*, using basic results from linear regression analysis:

$$k^* = \left(\sum_{j=1}^{n}(C_j - \overline{C})(p_j - \overline{p})\right) / \sum_{j=1}^{n}(p_j - \overline{p})^2,$$

$$d^* = \overline{C} - k^*\overline{p},$$

where $\overline{p} = (1/n)\sum_{j=1}^{n} p_j$ is the mean processing time, and $\overline{C} = (1/n)\sum_{j=1}^{n} C_j$ is the mean completion time.

Kahlbacher and Cheng (1995) formulated the combined sequencing and due date assignment problem as a pure sequencing problem, considering the optimal parameter values k and q implicitly; i.e., if the job sequence is fixed, the optimal due date parameters can be computed as functions of the job sequence. The objective function for the pure sequencing problem is:

$$Var(C)Var(p) - Cov(C,p)^2,$$

where $Var(p) = (1/n)\sum_{j=1}^{n}(p_j - \overline{p})^2$ denotes the variance of the processing times, $Var(C) = (1/n)\sum_{j=1}^{n}(C_j - \overline{C})^2$ denotes the variance of the completion times, and $Cov(C,p) = (1/n)\sum_{j=1}^{n}(C_j - \overline{C})(p_j - \overline{p})$ denotes the covariance of the processing times and the job completion times.

Kahlbacher and Cheng (1995) showed that $Cov(C,p)$ is monotonically decreasing with $\overline{C}$ and that the objective function can be seen as a combination of two conflicting components, the mean completion time $\overline{C}$ and the completion time variance $Var(C)$. They proposed a heuristic approach to solve this special bicriterion problem. This approach is based on a linearization of the non-linear bicriterion objective function, and leads to an $O(n\log n)$ heuristic algorithm. The optimal sequence is shown to be a V-shaped sequence.

Note that the considered problem is NP-hard since the first part of the objective function $Var(p)Var(C)$ is minimized by a sequence that minimizes the completion time variance. The completion time variance (CTV) problem is known to be NP-hard (Kubiak, 1993) and can be solved by a pseudopolynomial-time algorithm (De, Ghosh and Wells, 1992).

Gupta and Sen (1983) and Szwarc (1990) proposed branch-and-bound procedures for solving the MSD problem with distinct due dates, but these procedures are effective for the problems up to 10 jobs in size.

Assume that the jobs are numbered in non-decreasing order of their processing times (SPT order). Lee (1991) showed that, for the MSD problem with PPW due dates, the SPT sequence is optimal if $-kp_1 \le q \le (p_1 + p_2)/2$.

Lee (1991) investigated a worst-case performance of the SPT heuristic for the MSD problem with PPW due dates, and proved that the following properties hold.

Property 7. If $q > (p_1 + p_2)/2$, then $f(SPT) - f(OPT) \le n^2 q(p_n - p_1)/2$, where $f(SPT)$ is the MSD for SPT sequence, and $f(OPT)$ is the optimal value of MSD.

Property 8. If $q > (p_1 + p_2)/2$ and $p_j = p_1(j-1)\delta$ for $j=2,\dots,n$, $\delta > 0$, then $f(SPT) - f(OPT) \le n^2(n^2-1)q\delta/3$ and

$$\lim_{n\to\infty} \frac{n^2(n^2-1)q\delta/3}{f(SPT)} = 0.$$

The last property shows that the SPT sequence is asymptotically optimal if there is an equal increment between successive processing times.

3.2. Assume, as in **3.1**, that n simultaneously available jobs are processed on a single machine without the machine idle time, and that due dates are set as $d_j = kp_j + q, j=1,\dots,n$.

Kahlbacher and Cheng (1995) considered the problem of finding the values of k and q and an optimal sequence which minimize the maximum absolute lateness

$$\max_{1\le j\le n}|L_j| = \max_{1\le j\le n}|C_j - d_j|.$$

They showed that the following property is valid.

Property 9. If $0 \leq k \leq 1$, then the objective function achieves its minimum over all sequences and all parameter values of k and d at $k = 1$.

Hence, the values for k smaller than 1 are always dominated by values greater than or equal to 1.

Kahlbacher and Cheng (1995) showed that an optimal job sequence is given by the SPT sequence, if the parameter k is larger than or equal to 1. Since, by virtue of Property 9, an optimal parameter value for k is such that $k^* \geq 1$, the SPT sequence is optimal for the general problem with $k \geq 0$.

The problem of determining the optimal parameters k and q was reduced to a linear programming problem with three variables and $2n$ constraints (Kahlbacher and Cheng, 1995). Cheng and Kovalyov (1998) showed that the optimal parameters k and q can be found by a graphical approach in $O(n \log n)$ time.

For parallel identical machines, the problem of finding the values of k and q and an optimal schedule which minimize the maximum absolute lateness

$$max_{1 \leq j \leq n}|L_j| = max_{1 \leq j \leq n}|C_j - d_j|$$

was considered by Cheng and Kovalyov (1998). They showed that the problem is NP-hard if the number of machines $m \geq 2$ is a constant, and it is strongly NP-hard if m is a variable. The strong NP-hardness of the problem with a constant number of machines remains an open question.

3.3. Assume that a partially ordered set N of jobs with given release dates is to be processed on a single machine under an assumption that preemption is allowed.

For the PPW due dates given as $d_j = kp_j + q$, $k \geq 0$, $q \geq 0$, Cheng and Gordon (1994) considered the problem of finding an optimal schedule S^* and optimal values k^* and q^* that jointly minimize the objective function

$$f(k,q,S) = \Phi(k,q) + \alpha\, max_{j \in N} T_j,$$

where $\alpha > 0$, and $\Phi(k,q) \geq 0$ is a non-decreasing convex function of k and q with $\Phi(0,0) = 0$.

For this problem, the property similar to Property 6, holds.

Property 10. The schedule S^* obtained by applying Algorithm GT-BLLR (see **1.1**) is optimal independently of the values of k and q.

Cheng and Gordon (1994) showed that there exists a point (k^*, q^*), where the objective function $f(k,q,S^*)$ achieves a minimum on the domain $k \geq 0$, $q \geq 0$, and any point of local minimum is a point of global minimum for this function. Therefore, any convex optimization technique (see, for example, Himmelblau, 1972, and Polak, 1997) may be used to find k^* and q^*.

In particular, one may fix, say k, and find an optimal value of q with respect to this k. Denote this value as $q^*(k)$. Then, the required value of k^* may be found by minimizing $f(k, q^*(k), S^*)$ using some known method of one-dimensional minimization of convex functions. Finally, set $q^* = q^*(k^*)$.

Cheng and Gordon (1994) presented a simple graphical approach for the problem with objective function

$$f(k,q,S) = \Phi(k,q) + \alpha \max_{j \in N} T_j,$$

where $\Phi(k,q) = \Phi_1(k) + \gamma q$, $\Phi_1(k)$ is a non-decreasing convex function with $\Phi_1(0) = 0$ and $\gamma \geq 0$. This approach uses Cheng's (1991) results for the TWK due dates.

The results for TWK, NOP and PPW due date models are summarized in Table 2, where we use the same notations as in Table 1. The notations $d_j = kp_j$, $d_j = kp_j^m$, $d_j = kM_j$ and $d_j = kp_j + q$ denote, respectively, TWK, TWK-power, NOP and PPW due dates. The notation *Pm* denotes the problem with *m* parallel identical machines when *m* is a constant, while the notation *P* denotes parallel identical machine problem with variable *m*. The remark *for schedule* in the **Algorithm** column shows that the complexity is indicated for finding an optimal schedule but not optimal parameters, while the remark *for k and q* shows the complexity of finding the optimal values of parameters *k* and *q*. The notation Enumer. denotes enumerative procedure, and LP denotes linear programming algorithm.

Table 2. TWK, NOP and PPW due date assignment and scheduling

Problem	Complexity	Algorithm
$1\|d_j = kp_j\|\sum L_j^2$	P	$O(n\log n)$ Cheng (1984)
$1\|d_j = kp_j^m\|\sum C_j + \sum L_j^2$	P	$O(n\log n)$ (for schedule) Cheng (1987), Cheng&Li (1989)
$1\|d_j = kM_j\|\gamma k + \max T_j$	P	$O(n\log n)$ Alidaee (1992)
$1\|d_j = kp_j\|\Phi(k) + \alpha \max T_j$	P	$O(n\log n)$ Cheng (1991)
$1\|pmtn, prec, r_j, d_j = kp_j^m\|\Phi(k,m) + \alpha \max T_j$	P	$O(n^2)$ (for schedule) Cheng&Gordon (1994)
$1\|d_j = kp_j^m\|\sum(\gamma k + \|L_j\|)$	P	$O(n\log n)$ (for k) Cheng, (1987b)Van de Velde (1990)
$1\|d_j = kp_j + q\|\sum L_j^2$	NP-hard (from Kubiak, 1993)	Heurist. Lee (1991), Kahlbacher&Cheng(1995) Enumer. Gupta&Sen (1983), Szwarc (1990)
$1\|d_j = kp_j + q\|\max_j\{\|L_j\|\}$	P	$O(n\log n)$ (for schedule) Kahlbacher&Cheng(1995) LP (for *k* and *q*) Kahlbacher&Cheng (1995) $O(n\log n)$ Cheng&Kovalyov (1998)
$1\|pmtn, prec, r_j, d_j = kp_j + q\|\Phi(k,q) + \alpha \max T_j$	P	$O(n^2)$ (for schedule) Cheng&Gordon (1994)
$1\|pmtn, prec, r_j, d_j = kp_j + q\|\Phi(k) + \gamma d + \alpha \max T_j$	P	$O(n^2)$ Cheng&Gordon (1994)
$Pm\|d_j = kp_j + q\|\max_j\{\|L_j\|\}$	NP-hard Cheng& Kovalyov (1998)	
$P\|d_j = kp_j + q\|\max_j\{\|L_j\|\}$	Strongly NP-hard Cheng& Kovalyov (1998)	

4. POSITIONAL AND OTHER DUE DATES

In this section, we consider scheduling problems in the case when the set of due dates is given, and the problem consists in assigning the jobs to the due dates.

In a class of generalized due date scheduling problems recently identified by Hall (1986) and Hall, Sethi and Sriskandarajah (1991), the due dates are specified according to the position in which a job is completed, rather than the identity of that specific job. When k jobs should be completed by the kth due date, these due dates are referred to as positional due dates (Hoogeveen, Lenstra and van de Velde, 1997). For instance, let three due dates d_1, d_2, d_3, where $d_1 < d_2 < d_3$, be given for the set of three jobs. The scheduling objective is to complete at least one job by d_1, at least two jobs by d_2, and all three jobs by d_3. This method of due date assignment describes the situation where what matters is *how many* jobs have been completed by any point in time, rather than *which* jobs they are.

Hall (1986) and Hall, Sethi and Sriskandarajah (1991) described several applications in petrochemical industry, in public utility planning and survey design, as well as in flexible manufacturing systems, where a customer places no priority on the jobs being performed, and a number of jobs must be completed by certain dates without regard to which of these jobs. As consequence, positional due dates occur naturally in these applications.

Throughout this section, we refer to the due dates defined in the traditional (job specific) manner as *specific due dates*.

Let d_j^p denote the date by which j jobs should be completed. In the positional due date version, d_j^p is a due date of the jth job to be completed. It is assumed that $d_i^p \le d_k^p$ for all $1 \le i < k \le n$.

Given the set of due dates and, independently, the set of jobs, we may also consider the situation, when the jobs can be assigned to the due dates using some rule, without taking into account their position in the schedule. Let us denote such due dates by d_j^a for each job j. The problems with due dates d_j^a are considered in **4.2** below.

4.1. Consider scheduling problems with positional due dates. Let $C_{[j]}$ denote the completion time of the jth job completed. Then, $T_{[j]} = \max\{0, C_{[j]} - d_j^p\}$ is the tardiness of this job, $L_{\max} = \max_j \{C_{[j]} - d_j^p\}$ is the maximum lateness of jobs, and $\sum_{j=1}^n U_{[j]}$ is the number of late jobs, where $U_{[j]} = 0$ if $C_{[j]} \le d_j^p$, and $U_{[j]} = 1$ otherwise.

Let us first consider the single machine problems.

Hall (1986) showed that, for the positional due dates, SPT sequence is optimal for both the problem of minimizing the total tardiness ($\sum T_{[j]}$) and the problem of minimizing the number of late jobs. Note, that the first problem for the specific due dates is NP-hard (Du and Leung, 1990), while the second one is solvable in $O(n \log n)$ time (Moore, 1968).

The problems of minimizing $\sum T_{[j]}$ or $\sum U_{[j]}$ with given release dates are shown to be strongly NP-hard (respectively, by Hall, 1986, and Hall, Sethi and Sriskandarajah, 1991) just as they are for the specific due dates. The problems of minimizing the weighted number of late jobs ($\sum w_{[j]}U_{[j]}$) or total weighted tardiness ($\sum w_{[j]}T_{[j]}$) are shown to be NP-hard (respectively, by Hall, 1986, and Sriskandarajah, 1990). Note that the latter problem for the specific due dates is strongly NP-hard (Lawler, 1977; Lenstra, Rinnooy Kan and Brucker, 1977).

Hall, Sethi and Sriskandarajah (1991) proposed an $O(n \log n)$ algorithm for the problem which consists in minimizing the maximum lateness with given release dates when preemption is allowed. The algorithm is a simple extension of SPT rule: at each decision point (whenever a job is released or a job is completed) a job with the shortest remaining processing time is chosen to be processed next (among the available jobs, which are released but their processing is not yet completed). This rule may be called SRPT (shortest remaining processing time). When preemption is not allowed, the problem is strongly NP-hard (Hall, Sethi and Sriskandarajah, 1991).

Chu (1996) defined a new class of scheduling criteria called over-regular criteria: $f(\sigma)$ is over-regular if, for any pair of schedules σ and σ' such that $C_{[j]}(\sigma) \leq C_{[j]}(\sigma')$ for all $j=1,\dots,n$, we have $f(\sigma) \leq f(\sigma')$. Some criteria involving positional due dates are over-regular, and among them the objective functions for the problems of minimizing the maximum lateness, the number of late jobs, and the total tardiness. If jobs are preemptive, any over-regular criteria problem with given release dates can be polynomially solved by the SRPT rule (Chu, 1996). By proving some dominance rules and using a lower bounding procedure with preemption, Chu (1996) proposed a branch and bound algorithm capable of solving instances with up to 70 jobs for the non-preemptive problem of minimizing total tardiness.

For the problems with given precedence constraints and unit processing times, any sequence satisfying the precedence constraints is optimal for the objective functions $\sum T_{[j]}$ or

$\sum U_{[j]}$ (Hall, Sethi and Sriskandarajah, 1991), while for the specific due dates these problems are NP-hard even for chain-like precedence constraints (Leung and Young, 1989; Lenstra and Rinnooy Kan, 1980).

Sriskandarajah (1990) showed that the problem of minimizing L_{max} with given precedence constraints is strongly NP-hard even if the precedence relation is chain, and this problem remains strongly NP-hard when release dates are given and preemption is allowed. As a consequence, the similar problems with objective functions $\sum T_{[j]}$ and $\sum U_{[j]}$ are also strongly NP-hard. It is interesting to notice that the corresponding problems of minimizing L_{max} for specific due dates are polynomially solvable even for arbitrary precedence constraints (Lawler, 1973; Blazewicz, 1976; Baker *et al.*, 1983; Gordon and Tanaev, 1983).

Whenever it is desirable to give equal treatment to all jobs (customs), that is to make the lateness of all jobs as close to each other as possible, the problem of minimizing the difference between the maximum and minimum lateness arises. Let $L_{\min} = \min_j \left\{ C[j] - d_j^p \right\}$ be the minimum lateness of jobs, then $\Delta L = L_{\max} - L_{\min}$ is the range of lateness. Tanaka and Vlach (1997a) proved that the single machine problem of minimizing ΔL is NP-hard in the strong sense both with and without the requirement that machine idle time between consecutive jobs is forbidden. They proposed also an $O(n \log n)$ approximation algorithm for this problem with the performance ratio $\lceil n/2 \rceil$ between the optimal value of ΔL and the value found by the algorithm.

Some practical applications lead to the single machine problem with both positional and specific due dates. Tanaka and Vlach (1997b) describe the following example where such problem arises naturally. A computer dealer may face a situation to install the same computer systems to several companies, and should satisfy both requirements given by the provider and companies. The dealer is required to install five systems per month by a computer provider who does not care which systems are sold. This induces the positional due dates. However, the dealer is required to install each system by its due date specified by a company. The times needed to install a system can be different as well as the dates at which companies want to start using the system. This induces processing times and specific due dates.

Considering the problems with both positional and specific due dates, let us denote by L_j^s the lateness of job j according to specific due date d_j, and by L_j^p the lateness of job j according to positional due date d_j^p.

Tanaka and Vlach (1997b) proved the NP-hardness in the strong sense of the problems of minimizing $\max\{L_{\max}^p, -L_{\min}^s\}$ and $\max\{L_{\max}^s, -L_{\min}^p\}$, where $L_{\max}^p = \max_{1\le j\le n} L_j^p$, $L_{\min}^p = \min_{1\le j\le n} L_j^p$ and $L_{\max}^s = \max_{1\le j\le n} L_j^s$, $L_{\min}^s = \min_{1\le j\le n} L_j^s$. For the problem of minimizing $\max\{L_{\max}^s, L_{\max}^p\}$, they proposed an $O(n\log n\log A)$ algorithm, where

$$A = \max\{L_{\max}^p(EDD) - L_{\max}^p(SPT), L_{\max}^s(SPT) - L_{\max}^s(EDD)\},$$

and EDD and SPT denote, respectively, the schedules obtained by the earliest due date rule and the shortest processing time rule.

Let us consider now the multiple machine problems with positional due dates.

For m parallel identical machines, Hall, Sethi and Sriskandarajah (1991) showed that the problem of minimizing the maximum lateness can be polynomially solved if preemption is allowed. This is done by showing that any problem with positional due dates is equivalent to a problem with specific due dates, and the latter problem can be polynomially solved by the algorithm of Horn (1974). The transformation is done in the following way : the earliest due date is assigned to the job with the shortest processing time, the next earliest due date is assigned to the job with the next shortest processing time, etc. Note that Horn (1974) gave a procedure to solve the problem of minimizing $L_{\max}$ in $O(n^2)$ time, while Gonzanez and Johnson (1980) gave a more efficient $O(mn)$ algorithm.

For the non-preemptive case, the problems of minimizing maximum lateness or the total tardiness are NP-hard even for two identical machines (Hall, 1986).

Hall, Sethi and Sriskandarajah (1991) showed that the problems which consist in minimizing the maximum lateness (as well as minimizing the number of late jobs or the total tardiness) in flow shop, job shop and open shop are NP-hard even for two machines. The proof is done by a transformation from the 3-partition problem. In the same way, it is shown that these problems are NP-hard if preemption is allowed. Surprisingly, minimizing the maximum lateness for a preemptive two machine open shop model is strongly NP-hard when positional due dates apply, while it is polynomially solvable in a specific due date version even for m machines (Cho and Sahni, 1981).

4.2. Agnetis *et al.* (1997) and Gordon and Kubiak (1996) considered the single machine model when the set of release dates and the set of due dates are given, and the problem consists in assigning jobs with given processing times to the release and due dates. In this model, jobs are released at regular time intervals (cycle time T_c), and ideally each job should be completed within the cycle time. So, release date $(i-1)T_c$ and due date iT_c are associated with each cycle time interval i, $i=1,...,n$, (but not with a specific job). It is assumed that the average processing time is equal to the cycle time, and that the last job must be completed on time. The model has an application in the production lines where jobs are released at a constant time intervals. If, for the sake of simplicity, we normalize all job processing times to the cycle time T_c, the model under consideration will be the following.

A set $N=\{1,2,...,n\}$ of jobs is to be processed on a single machine. Job j has rational processing time p_j. It is assumed that $\sum_{j=1}^{n} p_j = n$, the machine can handle at most one job at a time, and all jobs have to be finished in the interval $[0,n]$. Let us denote by long and short jobs those jobs for which $p_j > 1$ and $p_j < 1$, respectively. The jobs with $p_j = 1$ can be disregarded, as we will see hereafter.

Instead of assigning jobs to the release and due dates, it is convenient to consider that each time unit with integer end points in $[0,n]$ is to be assigned to a job so that the initial point of the time unit be the release date of the job and the end of the unit be its due date.

Agnetis *et al.* (1997) considered the non-preemptive scheduling problem to minimize the total tardiness when release date $i-1$ and due date i are assigned to the job in ith position of a schedule, $i=1,...,n$.

Gordon and Kubiak (1996) considered both preemptive and non-preemptive scheduling problems of minimizing the number of late jobs, using two ways to assign release and due dates:

(1) release date $i-1$ and due date i are assigned to the job in ith position of a schedule, $i=1,...,n$, (for the preemptive case of the problem the job in ith position is the job with ith start time);

(2) release date $i-1$ and due date i, $i=1,...,n$, may be assigned to any job provided that no two jobs share the same release (due) date. In this case the release date of the job in *(i-1)*st position may be greater than the release date of the job in ith position of a schedule.

Let r_j^a and d_j^a denote, respectively, the release date and due date assigned to job *j*. We denote by C_j the completion time of job *j*, and by $C_{\max}$ the makespan. Let w_j be the weight of job *j*, and $U_j = 0$ if $C_j \le d_j^a$, otherwise $U_j = 1$. We denote by $[i]$ the job in *i*th position (the job with the *i*th start time) of a schedule.

Adopting the three-field notation $a|b|c$ for scheduling problems (Lawler *et al.*, 1993), we can use the notation $1|r_{[i]} = i-1, d_{[i]} = i, C_{\max} = n|\sum w_j T_j$ for the problem considered by Agnetis *et al.* (1997), and the following notations for the problems considered by Gordon and Kubiak (1996):

$1|r_{[i]} = i-1, d_{[i]} = i, C_{\max} = n|\sum w_j U_j$ (when preemption is forbidden, and release date $i-1$ and due date i are assigned to the job in *i*th position of a schedule, $i = 1,...,n$);

$1|pmtn, r_{[i]} = i-1, d_{[i]} = i, C_{\max} = n|\sum w_j U_j$ (the former problem when preemption is allowed);

$1|r_j^a = i-1, d_j^a = i, C_{\max} = n|\sum w_j U_j$ (when preemption is forbidden, and release date $i-1$ and due date i, $i = 1,...,n$, may be assigned to any job provided that no two jobs have equal release or due dates);

$1|pmtn, r_j^a = i-1, d_j^a = i, C_{\max} = n|\sum w_j U_j$ (the third problem with preemption allowed).

Notice that for all these problems, jobs with $p_j = 1$ can always be scheduled at the end or at the beginning of optimal schedules without increasing the value of objective function, and therefore these jobs can be disregarded.

Gordon and Kubiak (1996) proposed two $O(n)$ algorithms which are suitable for both $1|pmtn, r_{[i]} = i-1, d_{[i]} = i, C_{\max} = n|\sum w_j U_j$ and $1|pmtn, r_j^a = i-1, d_j^a = i, C_{\max} = n|\sum w_j U_j$ problems. The algorithms differ in the number of preemptions generated. For one of these algorithms, the maximal number of preemptions in the optimal schedule is $l+s-1$ when $l > 1$, and $s-1$ when $l = 1$ (here, l is the number of long jobs and s is the number of short jobs); for the other algorithm, the maximal number of preemptions is $s-1$. The first algorithm schedules short jobs in the end of the schedule, the second one schedules them as close as possible to the beginning of the schedule.

For the non-preemptive cases, the problems under consideration are shown to be NP-hard in the strong sense either for $\sum w_j T_j$ objective function (Agnetis *et al.*, 1997) or for $\sum w_j U_j$ objective function even with $w_j = 1$, $j = 1,...,n$, (Gordon and Kubiak, 1996). Agnetis *et al.* (1997) proposed a branch-and-bound algorithm (applicable for $n \le 18$) for the problem of minimizing the total weighted tardiness and gave a number of lower bounds to use in the branch-and-bound procedure.

4.3. Chand and Chhajed (1992) considered the single machine scheduling problem in the case when, for n jobs with given processing times, only vector $(n_1, n_2, ..., n_m)$ is given, where n_i is the number of jobs assigned to the ith due date, and m is the number of due dates; $\sum_{i=1}^{m} n_i = n$. The objective is to determine a schedule σ, the due dates $D_1 \le D_2 \le ... \le D_m$, and the sets of jobs I_i, $i = 1,...,m$, assigned to due date D_i, which jointly minimize the total penalty

$$F(D, I, \sigma) = \sum_{i=1}^{m} \sum_{j \in I_i} (\alpha E_j + \beta T_j + \gamma D_i),$$

where E_j and T_j are earliness and tardiness of job j, $D = \{D_1, D_2, ..., D_m\}$, $I = \{I_1, I_2, ..., I_m\}$, and α, β and γ are given non-negative constants.

Chand and Chhajed (1992) showed that the problem has an optimal solution with zero machine idle time, and therefore it is sufficient to consider permutation schedule σ to find an optimal solution. So, the objective function is similar to that considered by Panwalkar, Smith and Seidmann (1982) for the common due date (see **1.2** in Gordon, Proth and Chu, 1998).

The following property holds for the problem under consideration (Chand and Chhajed, 1992).

Property 11. For any given σ, there is an optimal D such that each D_i coincides with the completion time of a job in σ, and σ can be rearranged so that n_i consecutive jobs in σ are assigned to D_i.

Using this property, Chand and Chhajed (1992) proposed an $O(n \log n)$ algorithm to solve the problem. In the resulting optimal sequence, the first of each n_i consecutive jobs of I_i are in LPT order, while the other last jobs are in SPT order.

The particular cases when $m=1$ or $m=n$ were considered in Panwalkar, Smith and Seidmann (1982), and Seidmann, Panwalkar and Smith (1981), respectively (see **1.2** and **1.9** in Gordon, Proth and Chu, 1998).

Chand and Chhajed (1992) considered also the case when the vector $(n_1, n_2, \dots, n_m)$ is unknown (this vector is a decision variable together with *D*, *I* and σ). They showed that there is an optimal solution with $n_1 \geq n_2 \geq \dots \geq n_m$, and proposed a search algorithm to find an optimal n_1, n_2, ..., n_m. The algorithm generates vectors $(n_1, n_2, \dots, n_m)$ by partitioning n into m integer numbers such that $\sum_{i=1}^{m} n_i = n$ and $n_1 \geq n_2 \geq \dots \geq n_m \geq 1$. If R_m^n denote the total number of these vectors, the algorithm needs $O(R_m^n n \log n)$ time to find an optimal solution. Chand and Chhajed (1992) gave a computational procedure to find R_m^n.

The results of this Section are summarized in Table 3, where notation $d_{[j]} = d_j^p$ denotes positional due dates , notations $r_{[i]} = i-1$ and $d_{[i]} = i$ denote that release date $i-1$ and due date i are assigned to the job in ith position of a schedule, and notations $r_j^a = i-1$ and $d_j^a = i$ denote that release date $i-1$ and due date i, $i = 1, \dots, n$, may be assigned to any job provided that no two jobs have equal release or due dates. Notation $C_{\max} = n$ denotes the condition to finish all jobs in the interval $[0, n]$. Notation $d_j^a = D_i$, $\sum n_i = n$ denotes the problem considered in **4.3** with the vector $(n_1, n_2, \dots, n_m)$ of the numbers of due dates D_i to be chosen. As usual, the notations *pmtn*, *prec*, *chain* denote, respectively, the problem when preemption is allowed, the problem with precedence constraints given by an arbitrary acyclic graph, and by a chain-like graph. Notations *P2*, *F2*, *J2* and *O2* denote, respectively, the parallel identical machine, flow shop, job shop and open shop problems with two machines. Notation *P* denotes parallel identical machine problem with variable number of machines.

Table 3. Positional and other due date assignment and scheduling

Problem	Complexity	Algorithm
$1\|d_{[j]}=d_j^p\|\sum T_j$	P	$O(n\log n)$ Hall (1986)
$1\|d_{[j]}=d_j^p\|\sum U_j$	P	$O(n\log n)$ Hall (1986)
$1\|r_j,d_{[j]}=d_j^p\|\sum T_j$	Strongly NP-hard Hall (1986)	Enumer. Chu (1996)
$1\|r_j,d_{[j]}=d_j^p\|\sum U_j$	Strongly NP-hard Hall *et al.* (1991)	
$1\|d_{[j]}=d_j^p\|\sum w_jT_j$	NP-hard Sriskandarajah (1990)	
$1\|d_{[j]}=d_j^p\|\sum w_jU_j$	NP-hard Hall (1986)	
$1\|pmtn,r_j,d_{[j]}=d_j^p\|L_{\max}$	P	$O(n\log n)$ Hall *et al.* (1991)
$1\|pmtn,r_j,d_{[j]}=d_j^p\|\sum U_j$	P	$O(n\log n)$ Chu (1996)
$1\|pmtn,r_j,d_{[j]}=d_j^p\|\sum T_j$	P	$O(n\log n)$ Chu (1996)
$1\|r_j,d_{[j]}=d_j^p\|L_{\max}$	Strongly NP-hard Hall *et al.* (1991)	
$1\|prec,p_j=1,d_{[j]}=d_j^p\|\sum T_j$	P	$O(n^2)$ Hall *et al.* (1991)
$1\|prec,p_j=1,d_{[j]}=d_j^p\|\sum U_j$	P	$O(n^2)$ Hall *et al.* (1991)
$1\|chain,d_{[j]}=d_j^p\|L_{\max}$	Strongly NP-hard Sriskandarajah (1990)	
$1\|chain,pmtn,r_j,d_{[j]}=d_j^p\|L_{\max}$	Strongly NP-hard Sriskandarajah (1990)	
$1\|d_{[j]}=d_j^p\|L_{\max}-L_{\min}$	Strongly NP-hard Tanaka&Vlach (1997a)	Approx. Tanaka&Vlach (1997a)

Table 3. Positional and other due date assignment and scheduling (continuation)

Problem	Complexity	Algorithm
$P\|pmtn, d_{[j]} = d_j^p\|L_{\max}$	P	$O(mn)$ Hall *et al.* (1991)
$P2\|d_{[j]} = d_j^p\|c \in \{L_{\max}, \sum U_j\}$	NP-hard Hall (1986)	
$F2\|d_{[j]} = d_j^p\|c \in \{L_{\max}, \sum U_j, \sum T_j\}$	Strongly NP-hard Hall *et al.* (1991)	
$J2\|d_{[j]} = d_j^p\|c \in \{L_{\max}, \sum U_j, \sum T_j\}$	Strongly NP-hard Hall *et al.* (1991)	
$O2\|d_{[j]} = d_j^p\|c \in \{L_{\max}, \sum U_j, \sum T_j\}$	Strongly NP-hard Hall *et al.* (1991)	
$F2\|pmtn, d_{[j]} = d_j^p\|c \in \{L_{\max}, \sum U_j, \sum T_j\}$	Strongly NP-hard Hall *et al.* (1991)	
$J2\|pmtn, d_{[j]} = d_j^p\|c \in \{L_{\max}, \sum U_j, \sum T_j\}$	Strongly NP-hard Hall *et al.* (1991)	
$O2\|pmtn, d_{[j]} = d_j^p\|c \in \{L_{\max}, \sum U_j, \sum T_j\}$	Strongly NP-hard Hall *et al.* (1991)	
$1\|r_{[i]} = i-1, d_{[i]} = i, C_{\max} = n\|\sum w_j T_j$	Strongly NP-hard Agnetis *et al.* (1997)	Enumer. Agnetis *et al.* (1997)
$1\|r_{[i]} = i-1, d_{[i]} = i, C_{\max} = n\|\sum w_j U_j$	Strongly NP-hard Gordon&Kubiak (1996)	
$1\|pmtn, r_{[i]} = i-1, d_{[i]} = i, C_{\max} = n\|\sum w_j U_j$	P	$O(n)$ Gordon&Kubiak (1996)
$1\|r_j^a = i-1, d_j^a = i, C_{\max} = n\|\sum w_j U_j$	Strongly NP-hard Gordon&Kubiak (1996)	
$1\|pmtn, r_j^a = i-1, d_j^a = i, C_{\max} = n\|\sum w_j U_j$	P	$O(n)$ Gordon&Kubiak (1996)
$1\|d_j^a = D_i, \sum n_i = n\|\sum\sum(\alpha E_j + \beta T_j + \gamma D_i)$	P	$O(n \log n)$ Chand&Chhajed (1992)

5. CONCLUSION

In this paper, we consider various due date assignment methods for deterministic scheduling problems. Together with Gordon, Proth and Chu (1998), the paper provide a unified framework of the due date assignment and scheduling decisions for the static production settings. Most of the observed results are destined for the single machine, and the further research may be extended to the more complicated scheduling settings. Dynamic due date assignment models may be also a subject of further development (an example of the work in this direction is the paper of Cheng and Jiang, 1998, where dynamic TWK and PPW methods are proposed together with an improved dispatching rules and simulation results). Another promising direction is the research of the multicriteria due date assignment and scheduling problems where little is done up to now.

ACKNOWLEDGEMENT

This work was supported in part by INTAS (Project INTAS-98-820). The first author was also partly supported by ISTC (Project B-104) and by the Belarus Foundation for Basic Research (Project F97-147). The paper was prepared when the first author was visiting INRIA (Metz) and UTT (Troyes). Support of this visit from CIES and the University of Technology of Troyes is gratefully acknowledged.

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http://www.inria.fr
ISSN 0249-6399

RR-3537